Engineering Notes

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Control and Guidance Systems with Automatic Aperiodic Sampling

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IN a majority of servomechanism systems the error sensing is done in a continuous manner, but in some special applications such as space technology and some chemical processes, the error sensing is done by sampling at predetermined constant sampling time intervals, $\Delta t = (t_n - t_{n-1})$.[1,2] Sampling is acceptable when the values of the governing parameters change at a relatively slow rate or when the need for a corrective action is occurring at long time intervals. Such a system (with a predetermined constant sampling rate) can be improved by introducing variable aperiodic sampling time intervals. The purpose of variable sampling intervals can be the need for saving energy and weight (having only one sampling instead of, say, 1000 or 10,000), as in space applications. Such systems also reduce transients and hunting due to too frequent corrective control action.

The rules governing aperiodic sampling should be based on some reasonable ability to predict the future behavior of the system.[3] Obviously such predictions should be based on the past time history of the system. When disturbances of the system are of random nature, some statistical mathematical method of predicting would be in order. On the other hand, if the system is subject to smooth drifting errors (as in space vehicles), the prediction of its future behavior is relatively simple.

Following is an example illustrating the principles of such aperiodic sampling. For simplicity, assume that only one parameter, "Y," needs to be controlled. Let the desired value of this parameter be designated Y_{command}. If the value of the parameter at time t_n drifted to the value Y_{t_n} then the error is

$$e_n = Y_{\text{com}} - Y_{t_n} \tag{1}$$

Let the maximum acceptable error be e_{acp}. Then at the time when the error of the system reaches that maximum value, e_{acp}, the system has to be sampled and a corrective control must be initiated. How can this time be predicted? For the engineer who is accustomed to working with linear systems, it is most natural to base such a prediction on previous values of first and second derivatives of the error. With the help of Fig. 1 the following relationships can be established.

According to notation shown in Fig. 1 the errors are:

$$e_{t_{n-1}} = Y_{\text{com}} - Y_{t_{n-1}} \tag{2}$$

$$e_{t_{n-2}} = Y_{\text{com}} - Y_{t-2} \tag{3}$$

$$e_{t_{n-3}} = Y_{\text{com}} - Y_{t-3} \tag{4}$$

Then the first derivatives of the errors are:

$$\dot{e}_{t_{n-1}} = \frac{e_{t_{n-1}} - e_{t_{n-2}}}{t_{n-1} - t_{n-2}} = \frac{Y_{\text{com}} - Y_{t_{n-1}} - Y_{\text{com}} + Y_{t_{n-2}}}{t_{n-1} - t_{n-2}} = \frac{Y_{t_{n-2}} - Y_{t_{n-1}}}{t_{n-1} - t_{n-2}} \tag{5}$$

$$\dot{e}_{t_{n-2}} = \frac{e_{t_{n-2}} - e_{t_{n-3}}}{t_{n-2} - t_{n-3}} = \frac{Y_{t_{n-3}} - Y_{t_{n-2}}}{t_{n-2} - t_{n-3}} \tag{6}$$

The second derivative of the error is:

$$\ddot{e}_{t_{n-1}} = \frac{\dot{e}_{t_{n-1}} - \dot{e}_{t_{n-2}}}{t_{n-1} - t_{n-3}} \tag{7}$$

The predicted time interval for the new sampling is:

$$(\Delta t)_n = t_n - t_{n-1}$$

or

$$t_n = t_{n-1} + \Delta t$$

Assuming that at time t_{n-1} the error was driven to zero, the predicted acceptable error e_{acp} will be obviously determined by the following equation:

$$e_{acp} = \dot{e}(\Delta t) + \tfrac{1}{2}\ddot{e}_{t_{n-1}}(\Delta t)^2 \tag{8}$$

which causes a new sampling. Solving Eq. (8) for (Δt) we obtain

$$\Delta t = t_n - t_{n-1} = \frac{-\dot{e}_{t_{n-1}} + [(\dot{e}_{t_{n-1}})^2 + 2\ddot{e}_{t_{n-1}} e_{act}]^{1/2}}{\ddot{e}_{t_{n-1}}} \tag{9}$$

The time at which the next sample should occur, t_n, is:

$$t_n = t_{n-1} + \Delta t = t_{n-1} + \frac{-\dot{e}_{t_{n-1}} + [(\dot{e}_{t_{n-1}})^2 + 2\ddot{e}_{t_{n-1}} e_{act}]^{1/2}}{\ddot{e}_{t_{n-1}}} \tag{10}$$

The hardware of the proposed system requires:

1) a clock (solid-state oscillator)

2) memory storing:

$$\text{time}, Y_{\text{com}}, Y_{t_{n-1}}, Y_{t_{n-2}}, Y_{t_{n-3}}, e_{act}, t_{n-2}, t_{n-1}$$

(A hold circuitry could be employed when precision is not important.)

3) computer determining: $e_{t_{n-1}}$ [Eq. (2)], $e_{t_{n-2}}$ [Eq. (3)], $e_{t_{n-3}}$ [Eq. (4)], $\dot{e}_{t_{n-1}}$ [Eq. (5)], $\dot{e}_{t_{n-2}}$ [Eq. (6)], $\ddot{e}_{t_{n-1}}$ [Eq. (7)], time of next sampling t_n [Eq. (10)], and commanded control correction $(\delta_{\text{com}})_n$

4) sampler.

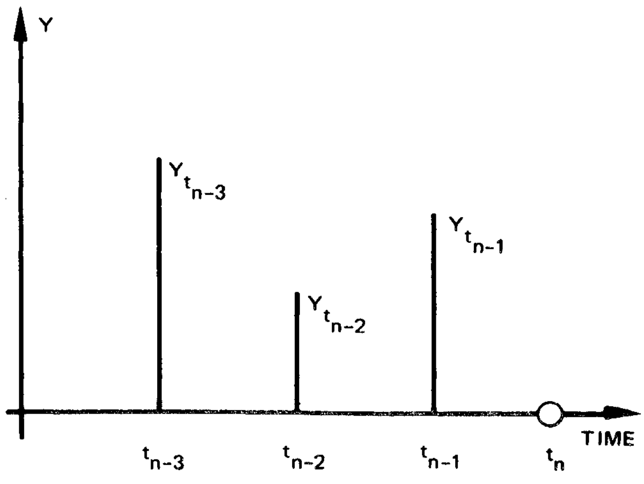

Fig. 1 Plot of the function vs time.

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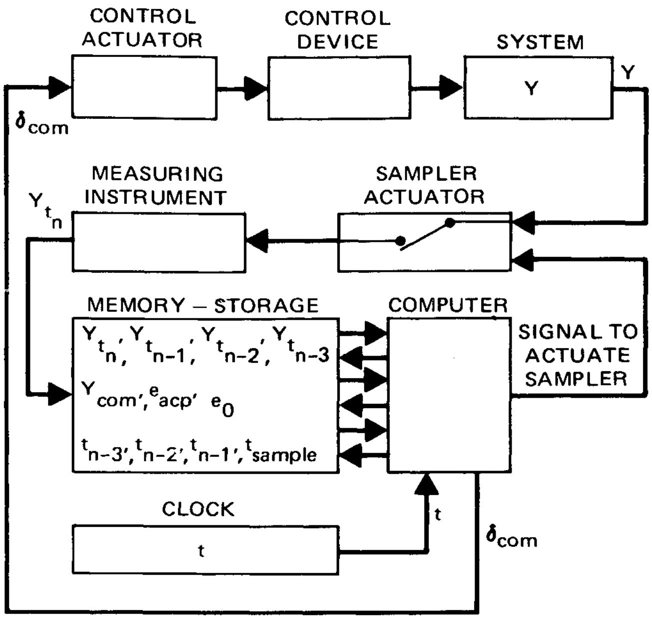

Fig. 2 Block diagram of the system.

The block diagram of the system is given in Fig. 2. It is possible to implement this system such as to provide a sampling interval, t_{sample}, during which the system works as a continuous system until the error e_n is driven to zero.

References

[1] Truxal, J. G., *Automatic Feedback Control System Synthesis*, McGraw-Hill, New York, 1955.

[2] Tou, J. T., *Digital and Sampled Data Control Systems*, McGraw-Hill, New York, 1959.

[3] Kalviste, J., "Analysis of Sampled Data System with Sampling Period a Function of the Error Signal," M.S. thesis, 1960, Dept. of Engineering, Univ. of Washington, Seattle, Wash.

Design of Neutral Burning Star Grains

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Nomenclature

A_{p_i} = initial port area
f = fillet radius
f/l = fillet ratio
n = number of star points
R_m = minimum inner radius of the star
S = burning perimeter
V_f = sliver volume
V_m = chamber volume
V_p = initial propellant volume
V_f/V_p = sliver fraction
w = neutral burning web thickness
y = burnt distance
ε = angular fraction
η_l = loading density, V_p/V_m
$\bar{\theta}$ = opening of neutral burning star points

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Introduction

CHARACTERISTICS of "straight through" internal burning star grains are discussed in Refs. 1–4. A typical segment of the neutral burning star grain is shown in Fig. 1. Vandenkerckhove and Barrère[1,2] consider l the characteristic dimension. For each number of star points considered, characteristic curves are presented in the form of two different plots, choosing the coordinates as S/l, $(y+f)/l$, A_p/l^2 and A_f/l^2 while ε is taken as the parametric quantity. Using these available plots, it is not easy to obtain a suitable grain geometry because: 1) two different plots are provided for each number of star points and it is required to go through a number of them to arrive at the suitable geometry, 2) the usually specified optimal quantities, the loading density and the sliver fraction, are not shown in the graphs, 3) angular fraction, ε which does not have any direct ballistic significance has been chosen as the parametric quantity. Stone[3] analyzes the internal burning star for the special case of $\varepsilon = 1$. The aim of the present work is to focus attention only on the neutral burning phase of the "straight through" star grains with wide variation in angular fraction and to present the characteristics in a convenient form for the grain design.

Evidently the current demands for high loading density and minimum sliver fraction imposed on the solid rocket grains have made the pure star grains of little use. Grain designs such as finocyl and slotted tubes have become more important and elaborate machine computations are increasingly being used. However, the present procedure, in addition to the academic interest, helps in a quick estimate of the grain configuration when the relatively poor performance of the star grains can be tolerated.

Characteristic Parameters

Usually in the design of grains, the basic propellant to be used with its approximate composition is assigned. For the known chamber pressure the burning rate of the propellant cannot be widely varied due to the limitations imposed by the other desirable ballistic and mechanical properties. Therefore, in addition to specifying the basic propellant, if neutral burning time and the optimum chamber pressure or the range of possible chamber pressures (usually this is narrow) are specified it comes to the point of designing the grain for the approximately known web thickness. So the neutral burning web thickness w is chosen as the characteristic dimension and the resulting characteristic ballistic parameters are w/l, S/w, and A_{p_i}/w^2. The other two additional parameters for the selection of grain configuration are loading density, η_l and sliver fraction, V_f/V_p. For the given thrust level, chamber pressure, propellant, and web thickness A_{p_i}/w^2

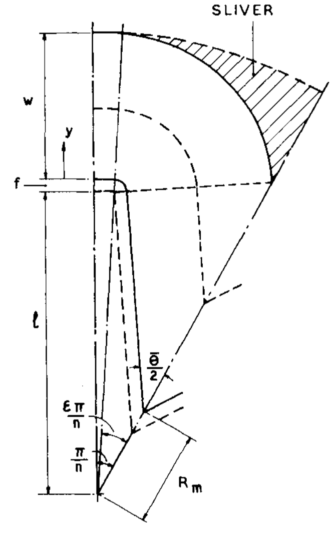

Fig. 1 Neutral burning star configuration.